Summary of Model Predictions for U_{e3} [*]

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Abstract

We present a short discussion on the expected magnitude of $|U_{e3}|$ in the context of various scenarios proposed to describe neutrino masses and mixing. Generic expectation is relatively large (> 0.05) values for $|U_{e3}|$ which occur in many well-motivated theoretical scenarios and models.

1 Introduction

The neutrino oscillations seen in the atmospheric and the solar neutrino experiments have been used to obtain information on the neutrino mixing matrix U which is now fairly well-known [1]. This review is aimed at providing a short summary of the theoretical expectations on the magnitude of one of the elements of U, namely U_{e3}.

The U_{e3} controls the strength of CP violation in neutrino oscillations. It also leads to important sub-leading effects in the solar and the atmospheric neutrino oscillations. It would therefore play an important role in deciding feasibility of the planned experiments [1] which look for these effects. It is thus important to have a rough theoretical estimate for $|U_{e3}|$ and the existing literature is full of this [2, 3]. While it is not possible to summarize all attempts in this short review, we concentrate on giving basic scenarios which lead to definite expectations on $|U_{e3}|$ illustrating them with specific

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models when appropriate. The basic message which emerges from this study is that there exists very large class of well-motivated theoretical scenarios within which relatively large (> 0.05) values for $|U_{e3}|$ are possible. In some specific scenarios, the present day phenomenology provides a lower bound on U_{e3} which is close to the existing upper bound making us think that discovery of a non-zero $|U_{e3}|$ is just round the corner!

Elements of U entering the description of neutrino oscillations are determined by the matrices U_l and U_ν which diagonalize the charged lepton and the neutrino mass matrices $\mathcal{M}_l$ and $\mathcal{M}_\nu$ respectively, $U = U_l^\dagger U_\nu$. One can always choose the standard CKM form for U_ν. The most general U_l then depends [4] on three angles θ_{ijl}, two Majorana and three Dirac phases in such a way that the U has the MNS form [2]. The U is thus determined by 12 different parameters in general.

In spite of the above complexity, one could have meaningful predictions for $|U_{e3}|$ using (i) simplifying theoretical assumptions such as quark lepton symmetry (ii) educated guesses following from the existing knowledge on neutrino oscillations and/or using (iii) some flavour symmetry. Unlike other angles, $|U_{e3}|$ is small; the combined analysis of the CHOOZ and the atmospheric data imply $|U_{e3}| \leq 0.26$ at 3σ. It then makes sense to start with neutrino mass textures which lead to zero $|U_{e3}|$ and then use some perturbations to obtain estimates for $|U_{e3}|$. We will systematically follow this approach. Before doing this, let us see what are generic expectations for $|U_{e3}|$. These expectations are based on a natural assumptions that the charged lepton mixing angles θ_{ijl} are determined by the corresponding lepton mass ratios, e.g. through square root formula. This implies

$$\theta_{13l} << \theta_{12l} \sim \sqrt{\frac{m_e}{m_\mu}} \; .$$

Since $U = U_l^\dagger U_\nu$, one gets

$$|U_{e3}| \approx |s_{13\nu} - s_{12l} \sin \theta_A| \; , \tag{1}$$

where θ_A is the atmospheric mixing angle. In the absence of any cancellations one finds that $|U_{e3}|$ is at least

$$|U_{e3}| \approx \mathcal{O}(\frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}}) \approx 0.05$$

which is near the projected detectable value in the long baseline experiments with super beams [1]. While this generic value gets realized in several models [5, 6], substantial deviations from it are possible since both $\theta_{13\nu}$ and θ_{12l} contribute to eq.(1) and these contributions may get added or subtracted.

2 Leptonic mass structures with zero U_{e3}

A systematic study of perturbations over structures which give zero $|U_{e3}|$ can give us some idea on possible ranges which $|U_{e3}|$ can take. With this view in mind, we list below various theoretical structures for the leptonic mass matrices leading to zero $|U_{e3}|$. The zero $|U_{e3}|$ may be attributed [7, 8, 9] to specific texture of the charged lepton mass matrix $\mathcal{M}_l$ (with almost diagonal $\mathcal{M}_\nu$) or to the specific choice of $\mathcal{M}_\nu$ (with almost diagonal $\mathcal{M}_l$). We consider both these cases. In the latter case, our choices for the structures of $\mathcal{M}_\nu$ are such that the atmospheric neutrino mixing and mass scale arise at the zeroth order in all of them.

2.1 Normal Hierarchy

$$\text{(A)}\quad \mathcal{M}_\nu = m \begin{pmatrix} 0 & 0 & 0 \\ 0 & s^2 & sc \\ 0 & sc & c^2 \end{pmatrix} \tag{2}$$

Here s(c) denotes the sine (cosine) of the atmospheric neutrino mixing angle θ_A and m^2 corresponds to $\Delta_{\mathbf{atm}}$. The solar scale and mixing angle are absent at the zeroth order. The above structure describes the normal neutrino mass hierarchy with $m_{\nu_3} \gg m_{\nu_{1,2}}$. The standard seesaw picture can generate the above texture when contribution of only one right handed neutrino dominates the seesaw mechanism [10].

2.2 Inverted Hierarchy

One can write two dominant structures corresponding to the inverted mass hierarchies $(m_{\nu_1} \approx m_{\nu_2} \gg m_{\nu_3})$ for the neutrinos:

$$\text{(B1)}\ \mathcal{M}_\nu = m \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & s^2 & sc \\ 0 & sc & c^2 \end{pmatrix} \qquad \text{(B2)}\ \mathcal{M}_\nu = m \begin{pmatrix} 0 & c & s \\ c & 0 & 0 \\ s & 0 & 0 \end{pmatrix} \tag{3}$$

Theoretically, texture $B2$ arise naturally from the $L_e - L_\mu - L_\tau$ symmetry [2]. This structure generates maximal solar mixing at the zeroth order although $\Delta_\odot = 0$ in case of both $(B1)$ and $(B2)$.

2.3 $\mu - \tau$ **symmetric structure**

The following is a more complex structure giving zero $|U_{e3}|$:

$$\mathcal{M}_\nu = \begin{pmatrix} a & b & b \\ b & d & e \\ b & e & d \end{pmatrix} \tag{4}$$

The above structure arise [11] from imposition of the $\mu - \tau$ interchange symmetry on $\mathcal{M}_\nu$. Unlike the previous textures, the neutrino spectrum in this case can be normal, inverted or quasi degenerate depending upon the values of the parameters in eq.(4). The atmospheric mixing angle is implied to be maximal while the solar mixing angle is arbitrary.

2.4 **Zero** $|U_{e3}|$ **from** M_l

There are two interesting textures for the charged leptons which lead to zero $|U_{e3}|$ if the neutrino mass matrix is diagonal. These are

$$\text{(C1) Democratic:} \quad \mathcal{M}_l = m_l \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{5}$$

$$\text{(C2) Lopsided} \quad \mathcal{M}_l = m_l \begin{pmatrix} 0 & 0 & \rho' \\ 0 & 0 & \rho \\ 0 & \epsilon & 1 \end{pmatrix} \tag{6}$$

with $\rho \sim \rho' \sim \mathcal{O}(1); \epsilon \ll 1$. The democratic structure [6, 12] may be argued to arise from an $S_3 \times S_3$ symmetry. The lopsided [7] structure can be nicely embedded in grand unified theory and has been extensively studied in this context. There exists more examples of structures with zero $|U_{e3}|$ [13]. More generally, vanishing of $|U_{e3}|$ can be understood [14] as a consequence of the invariance of the neutrino mass matrix in flavour basis under a class of discrete Z_2 symmetries which can in fact be used to characterize all possible textures with zero $|U_{e3}|$. Alternative possibility is the absence of any specific texture or symmetry in $\mathcal{M}_\nu$. This possibility termed as anarchy [15]. generally tends to give large mixing angles with a lower bound ≥ 0.1 on $|U_{e3}|$.

3   Perturbations and non-zero $|U_{e3}|$

The above discussed basic structures can be perturbed by adding small parameters in $\mathcal{M}_\nu, \mathcal{M}_l$ or in both. The predicted $|U_{e3}|$ differ considerably in these two cases. In many cases, perturbations to $\mathcal{M}_\nu$ generate both the solar scale and $|U_{e3}|$. This leads to correlations

$$|U_{e3}| \approx \sqrt{\frac{\Delta_\odot}{\Delta_{\text{atm}}}} \approx 0.2 \ \ \text{OR} \ \ |U_{e3}| \approx \frac{\Delta_\odot}{\Delta_{\text{atm}}} \approx 0.04$$

3.1   Perturbation to A

The perturbed texture (A) may be written as

$$(A') \quad \mathcal{M}_\nu = m \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & s^2 + \epsilon_{22} & sc + \epsilon_{23} \\ \epsilon_{13} & sc + \epsilon_{23} & c^2 + \epsilon_{33} \end{pmatrix} . \tag{7}$$

Without fine tuning or cancellations, the above structure can be shown [3] to lead to

$$|U_{e3}| \approx \mathcal{O}(1) \sqrt{\frac{\Delta_\odot}{\Delta_{\text{atm}}}} \approx 0.2$$

Thus various models based on this simple texture tend to predict rather large values for $|U_{e3}|$. Let us take an example

$$\mathcal{M}_\nu = m \begin{pmatrix} 0 & 0 & \lambda \\ 0 & s^2 & sc \\ \lambda & sc & c^2 \end{pmatrix} \tag{8}$$

The small perturbation λ simultaneously leads to large solar mixing angle, solar scale and $|U_{e3}|$

$$\tan 2\theta_\odot \sim -\frac{2s}{\lambda c^2} \ ; \ \ \frac{\Delta_\odot}{\Delta_{\text{atm}}} \sim 2\lambda^3 sc^2 \ ; \ \ |U_{e3}| \sim c\lambda \ .$$

Correlations among these imply a λ independent relation

$$|U_{e3}| = \frac{\tan 2\theta_\odot}{2 \tan \theta_A} \left(\frac{\Delta_\odot}{\Delta_{\text{atm}}} \cos 2\theta_\odot \right)^{1/2} \approx 0.13 \tag{9}$$

This relation [16] is more general than its derivation presented here. All neutrino mass textures [17] with $(\mathcal{M}_\nu)_{11} = (\mathcal{M}_\nu)_{12} = 0$ imply[1] the above relation and hence lead to quite large $|U_{e3}|$. Correlations between zeros in

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[1] If $(\mathcal{M}_\nu)_{13} = 0$ instead of $(\mathcal{M}_\nu)_{12}$ then one gets an equivalent relation with $\tan \theta_A \rightarrow \cot \theta_A$ in eq.(9).

textures of the leptonic mass matrices and $|U_{e3}|$ have been systematically studied [17, 18]. Many "texture zero" analysis presented in the literature may be regarded as perturbations to textures (A) or (B) and lead to [18] values of $|U_{e3}|$ in the range $0.02 - 0.2$. Quite a few of them actually predict rather large values for $|U_{e3}|$ as implied by eq.(9).

The example of eq.(8) is realistic and arises in the minimal $SO(10)$ model [19] employing type II seesaw mechanism for neutrino mass generation. Because of the quark lepton unification, the λ gets related to the Cabibbo angle giving rather large $|U_{e3}| \sim \frac{0.2}{\sqrt{2}} \approx 0.14$.

There are many other examples in which $|U_{e3}|$ gets related to the parameters in the quark sector either due to imposition of some GUT symmetry [20, 21, 22, 23, 24, 25] or due to the assumed quark lepton symmetry [26, 27, 28]. The resulting predictions are given in the Table 1.

3.2 Perturbation to B2

One needs to introduce [13] rather large perturbations to the structure (B2) in order to induce the solar neutrino oscillations through the LMA solution. This requires introducing deviation of $\theta_\odot$ from the maximal value and generation of a non-zero solar scale. The perturbations which do this also tend to generate [29, 30] rather large $|U_{e3}|$ in many cases although there exists an example in which $B2$ can be perturbed to obtain the solar scale without generating any corrections to $|U_{e3}|$ [31].

The non-maximal $\theta_\odot$ can be generated by small leptonic mixing angles θ_{ijl}. This also generates $|U_{e3}|$ and if θ_{12l} dominates as in the quark sector then one finds [29, 30]

$$\tan^2 \theta_\odot = 1 - 4U_{e3} \ .$$

The current bound $\tan^2 \theta_\odot \leq 0.64$ implies $|U_{e3}| \geq 0.09$. Thus a large value of $|U_{e3}|$ is forced in this scenario.

The breaking of the $L_e - L_\mu - L_\tau$ symmetry in the charged lepton sector cannot induce the solar scale at the tree level but it can do so radiatively. It is found that the LMA solution cannot be radiatively generated in supersymmetric models with zero 13 mixing at a high scale. One needs sizable value [30] for this mixing to do this. As a result, these models predict [30] rather large values ≥ 0.1 for $|U_{e3}|$.

Assumptions	$|U_{e3}|$	Examples
Dominant Contribution from U_l	$\mathcal{O}(\sqrt{\frac{m_e}{m_\mu}}) \approx 0.05$	[5, 6]
Type-II seesaw mechanism	$\mathcal{O}(\sqrt{\frac{\Delta_\odot}{\Delta_{\mathbf{atm}}}}) \sim 0.2$	[19]
GUT, Family Symmetry	0.15	[20]
	0.014	[21]
	$\mathcal{O}(\theta_c)$	[22]
	0.24	[23]
Models with two right handed neutrinos	0.07 (0.01)	[24]([25])
Quark-Lepton symmetry	0.05	[26]
	0.04-0.18	[27]
	0.06-0.2	[28]
Corrected Bi-Maximal U_ν	≥ 0.1	[29]
Corrected Bi-Maximal U_l	0.02	[9]
Radiative $\Delta_\odot$ in MSSM	$\simeq 0.1$	[30]
Radiative $|U_{e3}|$,Degenerate spectrum	$\mathcal{O}(0.1)$	[32, 33]
Anarchy	$\geq \mathcal{O}(0.1)$	[15]
Randomly perturbed textures		
Normal Hierarchy	$\mathcal{O}(\sqrt{\frac{\Delta_\odot}{\Delta_{\mathbf{atm}}}}) \sim 0.2$	
Inverted Hierarchy	≤ 0.01	[35]
Effects beyond GUT scale	≤ 0.04	[36]

Table 1: Predicted $|U_{e3}|$ under different assumptions and some illustrative examples.

Alternative possibility is to assume zero $|U_{e3}|$ at a high scale. It can be generated along with the LMA solution in case of the standard model [32] or in the presence of some seed value for $\Delta_\odot$ [33]. The former gives small values for $|U_{e3}|$ while in the latter case one can get $|U_{e3}| \sim 0.1$ if neutrino spectrum is quasi degenerate and/or $\tan\beta$ is high.

3.3 Perturbations to C

Lopsided models [7] with texture (C2) lead to bi-large leptonic mixing. The perturbation to this structure due to U_ν lead to a non-zero $|U_{e3}|$. If U_ν has the CKM form then it provides [9, 34] a nice explanation for the empirical relation $\theta_\odot = \frac{\pi}{4} - \theta_C$. In this case one finds [9] rather small $|U_{e3}| \approx \sin\theta_\odot |V_{cb}| \approx 0.02$.

The democratic structure for $\mathcal{M}_l$ needs to be perturbed to obtain non-zero masses for the first two generations. This perturbation also generates corrections to U_l and hence a non-zero $|U_{e3}|$ which is related to the charged lepton masses. Depending on the type of perturbations, one can get relatively large [26], $\mathcal{O}(\sqrt{\frac{m_e}{m_\mu}}) \sim 0.07$, or small [12] $\mathcal{O}(\frac{m_e}{m_\mu}) \sim 0.005$ $|U_{e3}|$ in this scenario.

4 Conclusions

Analysis of various textures for the leptonic mass matrices giving zero $|U_{e3}|$ shows that perturbations to these textures tend to generate relatively large $|U_{e3}|$ in many examples although there exist several examples with small values for $|U_{e3}|$. Summary is given in Table 1.

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